



SCEGGS DARLINGHURST

TRIAL EXAMINATION 200/

Mathematics

Year 12

2/3 *Unit*

TIME ALLOWED: 3 hours (plus 5 minutes reading time)

This is a TRIAL PAPER only and does not necessarily reflect the content or format of the Higher School Certificate Examination for this subject.

ASSESSMENT WEIGHTING 40%

Directions to Candidates:

- Attempt all ten questions. All questions are of equal value.
- Ensure that your student number is on this paper.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Approved scientific calculators should be used. Mathematical templates and geometrical instruments may be used.
- Begin each question on a new page write your number at the top of each page.
- The table of standard integrals is printed on the last page.

Question 1. (12 Marks)

Marks

(a) Factorise:

(i)
$$4y^2 - 100$$

1

(ii)
$$k^3 - 8$$

1

(b) Rationalise
$$\frac{8}{3-\sqrt{5}}$$

2

(c) Solve
$$\frac{2x}{3} - \frac{x+1}{4} = 1$$

2

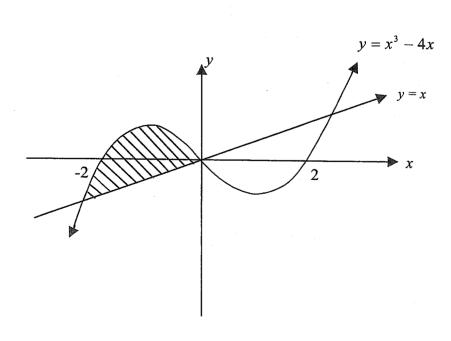
(d) Express 0.06 as a fraction in simplest form

2

(e) Solve
$$x^2 + 4x + 3 \ge 0$$

2

(f) Write a pair of inequalities to describe the shaded region:



Hence, find the area of AEBD

(h)

Question 2. (12 Marks) Start a new page. Marks Consider A(-6, -5) and B(8, 9)Calculate the distance AB, in simplest form (a) 1 Find the equation of the line joining A and B in general form (b) 3 (c) Find C, the midpoint of AB 1 Calculate the perpendicular distance of D (0, 3) from the line AB 1 Show that DC has the equation x + y - 3 = 0(e) 2 Write down the co-ordinates of E such that E lies on DC produced and DC = CE(f) 1 What sort of quadrilateral is AEBD? (g) 2 Justify your answer

Question 3. (12 Marks)

Start a new page

Marks

(a) 20 identical cards numbered 1 to 20 are placed in a bag, and one is drawn out at random.

2

What is the probability that this card will be less than 10 or divisible by 4?

(b) Differentiate:

$$(i) y = x^2 e^{3x}$$

2

(ii)
$$y = \frac{\sin 3x}{x}$$

2

(c) Find

(i)
$$\int \frac{dx}{x+5}$$

2

(ii)
$$\int \sec^2 5x \, dx$$

2

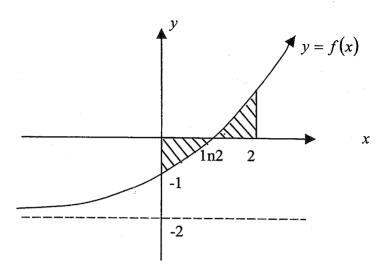
(d) Prove that
$$\frac{\sin \theta}{\cos \theta} (1 - \cot^2 \theta) + \frac{\cos \theta}{\sin \theta} (1 - \tan^2 \theta) = 0$$

Question 4 (12 Marks)

Start a new page

Marks

(a)



(i) Write an expression to describe the shaded area.

2

(ii) Give a possible equation for y = f(x)

1

(b) Evaluate
$$\sum_{k=2}^{6} 3^k$$

2

(c) Show that the equation of the tangent to the curve $y = \ln(x^2 + 3)$ at the point x = 3 is $x - 2y + \ln 144 - 3 = 0$

2

- (d) A function y = f(x) has a stationary point at (1, 2) and f''(x) = 6x 2Find:
 - (i) the equation of the function.

3

2

(ii) the co-ordinates of the other stationary point and determine its nature.

Question 5. (12 Marks)

Start a new page

Marks

3

(a) The table shows the values of a function f(x) for five x values.

	x	1	1.5	2	2.5	3
f	(x)	1.011	1.179	1.322	1.447	1.559

Approximate the value of $\int_{1}^{3} f(x) dx$ using the five values and Simpson's Rule.

(b) Find the values of m for which the equation $x^2 + 4mx + 8 - 4m = 0$ has equal roots

2

- (c) Consider the function $f(x) = x^3 12x + 16$
 - (i) Find the co-ordinates of any stationary points and determine their nature.

2

(ii) Find any points of inflexion

2

(iii) Sketch the curve $y = f(x), -5 \le x \le 3$

2

(iv) Find the minimum value of the curve in this domain.

Anna

Question 6. (12 Marks)

Start a new page

Marks

(a)

Consider the curve $(x-4)^2 = 16(y-2)$

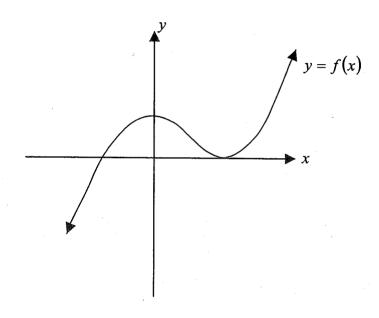
(i) Write down the co-ordinates of the focus.

2

(ii) State the equation of the directrix.

1

(b)



Copy this diagram onto your answer page.

(i) On the same set of axes, sketch y = f'(x)

1

(ii) On the same set of axes, using a dotted line, sketch a primitive of y = f(x)

2

- (c) The sum of the height, hcm, of a cylinder and the circumference of its base of radius rcm, is 10cm.
 - (i) Show that $h = 10 2\pi r$

1

(ii) Show that the volume of the cylinder is $V = \pi r^2 (10 - 2\pi r)$

1

- (iii) Find the exact value of r at which the volume of the cylinder is a maximum
- 3
- (iv) Hence find the maximum volume of the cylinder, correct to 2 decimal places.

American

Question 7. (12 marks)

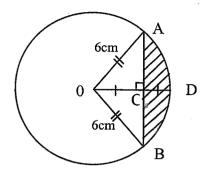
Start a new page

Marks

(a) Solve $e^{2x} - 4e^x - 5 = 0$.

3

(b)



0 is the centre of the circle. Find the exact value of the shaded area.

3

- (c) (i) Sketch the curve $y = -\sin(2x)$ and $y = \frac{x}{3}$ on the same axes in the domain $0 \le x \le \pi$
- 2
- (ii) Hence, state the number of solutions to the equation $-\sin(2x) = \frac{x}{3}$ in the domain $0 \le x \le \pi$
- (iii) One solution was approximated as $1 \cdot 9$. Use this approximation to calculate the area between the two functions in the domain $0 \le x \le 1 \cdot 9$ correct to 1 decimal place.

3

Question 8. (12 Marks)

Start a new page

Marks

(a) Sally and Brett decide to buy a new house for \$300000. They agree to pay back the loan in equal monthly instalments of P over 20 years.

Interest is charged at 6% p.a., compounded monthly.

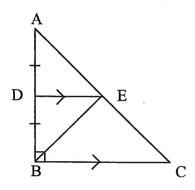
(i) Show that the amount Sally and Brett owe at the end of the second month is: $A_2 = 300000 (1.005)^2 - P (1 + 1.005)$

2

(ii) Hence, calculate the value of each monthly instalment, \$P.

3

(b) Triangle ABC has a right angle at B, D is the midpoint of AB, and DE||BC.



(i) Find the size of \angle ADE, giving reasons

1

(ii) Prove that $\triangle AED \equiv \triangle BED$

2

(iii) Prove that BE = EC

2

(c) Sketch a curve that has all of the following properties:

$$f(2)=1$$

$$f'(2) = 0$$

$$f''(2) = 0$$

$$f'(x) \ge 0$$
 for all real x

Question 9. (12 Marks)

Start a new page

Marks

(a) The percentage of the Mathematics course that is understood by the students in October is given by the equation:

$$P = \frac{-x^2}{1280} \left(4x - 240 \right)$$

where x is the number of revision assignments completed and P is the percentage of the course that is understood.

- (i) Show that a student completing 40 assignments understands 100% of the course.
- (ii) With reference to a graph, or otherwise, explain what happens when x > 40
- (iii) Find the value of x, $0 \le x \le 40$ for which the percentage of the course being understood increases at the greatest rate.
- (b) (i) On the same set of axes, sketch $y = e^x + 1$ and y = e + 1
 - (ii) Find the co-ordinates of the single point of intersection.
 - (iii) The area between y = e + 1, the y-axis and $y = e^x + 1$ is rotated about the x-axis.

Find the volume of the solid of revolution formed.

Question 10. (12 Marks)

Start a new page.

Marks

(a) A particle is moving in a straight line.

At time, t seconds, its distance, x metres, from a fixed point 0 is given by

$$x = 2 + 2\sin\frac{\pi}{4}t$$

(i) Calculate its initial position.

1

(ii) Find its exact velocity after 1 second.

2

(iii) Find the position of the particle when the particle is stationary for the second time.

2

(b) (i) Draw a table showing the outcome of rolling two dice and recording their sum.

1

(ii) Hence, copy and complete the following table of probabilities.

4

P (even number closer to 5 rather than 10)	
P (even number closer to 10 rather than 5)	
P (odd number closer to 5 rather than 10)	
P (odd number closer to 10 rather than 5)	

(iii) Year 2 students at SCEGGS play a game called Banko.

2

Two players, A and B, each begin with 18 blue counters and 12 red counters. The teacher rolls two dice and the sum is noted.

If the sum of the numbers is even, then Player A removes one of Player B's counters.

If the sum of the numbers is **odd**, then Player B removes one of Player A's counters.

The colour of the counter to be removed is determined as follows:

- a red counter is removed if the sum is closer to 5 than 10.
- a blue counter is removed if the sum is closer to 10 than 5.

The winner is the first person to remove all the other player's counters.

Use your results from part (ii) to explain whether or not this game is fair.

END OF PAPER

MATHEMATICS TRIAL SOLUTIONS, 2001

QUESTION 1: (12 marks)

(a) (i)
$$4(y^2-25)$$

= $4(y+5)(y-5)$

(ii)
$$(k-2)(k^2+2k+4)$$

(b)
$$\frac{8}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{24+8\sqrt{5}}{9-5}$$

= 6+2\sqrt{5}

(c)
$$\frac{2x}{3} - \frac{x+1}{4} = 1$$

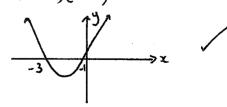
$$8x - 3(x+1) = 12$$

$$5x = 15$$

$$\cdot$$
, $x = 3$

(e)
$$x^2 + 4x + 3 > 0$$

 $(x+3)(x+i) > 0$



$$\therefore x \le -3$$
 and $x 7/-1$

(f)
$$y \le x^3 - 4x$$
 / Com and $y > x$ / 2

QUESTION 2: (12 marks)

(a)
$$AB = \sqrt{(8--6)^2 + (9--5)^2}$$

= $\sqrt{196 + 196}$
= $\sqrt{392}$

(b)
$$m = \frac{9-5}{8-6} = 1$$

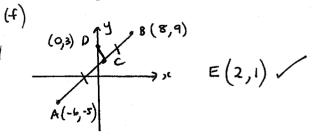
 $y - 9 = 1(2c-8)$
 $y = x + 1$
 $\therefore x - y + 1 = 0$

(d)
$$d = \frac{|0-3+i|}{\sqrt{1^2+1^2}}$$

= $\frac{2}{\sqrt{2}}$ $\sqrt{(=\sqrt{2})}$

(e)
$$m_{DC} = \frac{3-2}{0-1} = -1$$

 $\therefore y-3 = -1(x-0)$
 $x+y-3=0$



QUESTION 3: (12 marks)

$$(a) \frac{9+3}{20} = \frac{3}{5}$$

(b) (i)
$$u = x^2 v = e^{3x}$$

 $u' = 2x v' = 3e^{3x}$

$$\frac{dy}{dx} = 2xe^{3x} + 3x^{2}e^{3x}$$

$$= xe^{3x}(2+3x)$$

(ii)
$$u = \sin 3x \quad v = x$$

$$u' = 3\cos 3x \quad v' = 1$$

$$\frac{dy}{dx} = \frac{3x \cos 3x - \sin 3x}{x^2}$$

(c) (i)
$$\int \frac{dx}{x+5}$$

Calcut = $\ln(x+5) + C$

(d) LHS =
$$\frac{\sin \Theta}{\cos \Theta} \left(1 - \cot^2 \Theta\right) + \frac{\cos \Theta}{\sin \Theta} \left(1 - \tan^2 \Theta\right)$$

$$= \frac{\sin \Theta}{\cos \Theta} - \frac{\cos \Theta}{\sin \Theta} + \frac{\cos \Theta}{\sin \Theta} - \frac{\sin \Theta}{\cos \Theta}$$

QUESTION 4: (12 marks)

(a) (i) Area =
$$\int_{0}^{\ln 2} f(x) dx + \int_{h_{2}}^{2} f(x) dx$$
 (om 3

(ii)
$$y = e^{x} - 2$$
 (cm 3

(b)
$$\sum_{k=2}^{6} 3^{k} = 9 + 27 + 81 + 243 + 729$$

= 1089
$$\frac{9(1-3)}{1-3} = 1089$$

(c)
$$y = \ln(x^2+3)$$

 $\frac{dy}{dx} = \frac{2x}{x^2+3}$
At $x = 3$, $\frac{dy}{dx} = \frac{6}{12} = \frac{1}{2}$
At $x = 3$, $y = \ln 12$

$$y - h_{12} = \frac{1}{2}(x-3)$$

$$2y - 2h_{12} = x-3$$

$$3c - 2y + 2h_{12} - 3 = 0$$

$$x - 2y + h_{144} - 3 = 0$$

(d) (i)
$$f''(x) = bx-2$$

 $f'(x) = 3x^2-2x+C$
But (1,2) is stat point
... $0 = 3-2+C$
... $C = x^3$
... $f'(x) = 3x^2-2x-1$
... $f(x) = x^3-x^2-x+C$
(1,2) lies on when
... $C = 3$
... $C = 3$
... $C = 3$

(ii) Stat point => f'/x)=0
$3x^2-2x-1=0$
(3x +1)(x-1)=0
$y = 2 \approx \frac{100}{27} 86$
y = 2 ∞ 100 86 27

when
$$x = -\frac{1}{3}$$
, $f''(x) = -4$
. $(-\frac{1}{3}, \frac{68}{27})$ is a max +.p.

QUESTION 5: (12 marks)

(a)

l	1.5	2	2.5	3
1.011	1-179	1.322	1.447	1.559
1	4	2	4	ſ
1.011	4.716	2.644	5.788	1.559
	,		Σ=	15.718

(c) (i)
$$f'(x) = 3x^2 - 12$$

Stat. points occur $f'(x) = 0$ (alc
 $0 = 3x^2 - 12$ 7
 $\therefore x = \pm 2$
 $y = 0, 32$

$$f''(x) = 6x$$

when $x = 2$, $f''(x) = 12 > 0$
... $(2,0)$ is min + p.

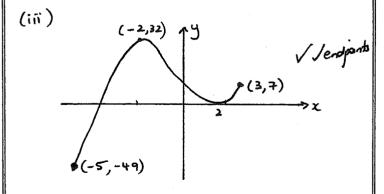
when
$$x=-2$$
, $f''(x)=-12 < 0$.: $(-2,32)$ is max +.p.

(ii) Possible p.o.c occur
$$f''(x) = 0$$

 $6x = 0$
 $y = 16$

Test:

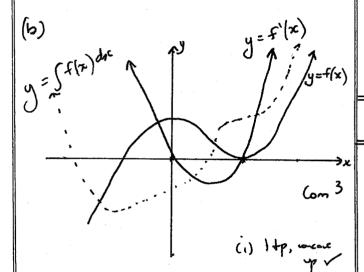
... (0,16) is a point of inflexion.



QUESTION 6: (12 marks)

- (a) (1) Vertex: (4,2)

 Focal leigth = 4
 - . Focus (4,6)
 - (ii) y = -2 /



(ii) horgatul 101 and min /

- (e) (i) $h + 2\pi r = 10$ $h = 10 - 2\pi r$
 - (ii) $V = \pi r^2 h$ = $\pi r^2 (10 2\pi r)$
 - (iii) $V = 10\pi r^2 2\pi^2 r^3$ $\frac{dV}{dr} = 20\pi r 6\pi^2 r^2$ $20\pi r 6\pi^2 r^2 = 0$ $2\pi r (10 3\pi r) = 0$ $\therefore r = 0 \text{ and } r = \frac{10}{3\pi}$

$$\frac{d^{2}V}{dr^{2}} = 20TT - 12TT^{2}r$$

$$When r = \frac{10}{3TT}, \frac{d^{2}V}{dr^{2}} < 0$$

$$\frac{r = \frac{10}{3TT}}{3TT} \text{ maximises volume.}$$

$$(iV) V = 10TT \left(\frac{10}{3TT}\right)^{2} - 2TT^{2} \left(\frac{10}{3TT}\right)^{3}$$

$$= 11.79 \left(2 \text{ ap}\right) V$$

QUESTION 7: (12 marks)

(a)
$$e^{2x} - 4e^{7} - 5 = 0$$

Let $u = e^{x}$

$$(u^{2} - 4u - 5 = 0)$$

$$(u - 5)(u + 1) = 0$$

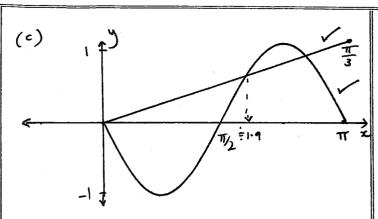
$$u = 5 \text{ or } -1$$

$$e^{x} = 5 \text{ or } e^{x} = -1$$

$$c = 105$$

$$c = 105$$

(b)
$$\frac{1}{6}$$
 $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{3}$ $\frac{1}{3}$



(iii)
$$A = \int_{0}^{1.9} \frac{x}{3} + \sin 2x \, dx$$

$$= \left[\frac{x^{2}}{6} - \frac{1}{2} \cos 2x \right]_{0}^{1.9}$$

$$= \left(0.602 + 0.395 \right) - \left(0 - \frac{1}{2} \right)$$

$$= 1.497$$

QUESTION 8: (12 marks)

(a) (i)
$$A_1 = 300,000 (1.005) - P$$

$$A_2 = A_1 (1.005) - P$$

$$= 300,000 (1.005)^2 - P(1+1.005)$$

(ii)
$$A_3 = A_3 (1.005) - P$$

= 300,000 (1.005) - P(1+1.005+1.05)

 $A_{n} = 300000 (1.005)^{2} - P(1 + 1.005 + ... + 1.005)$ But $A_{240} = 0$

: P= 300000 (1.005)240 :

= \$2149.29

: Sally + Brett pay back
each month.

(ii) DE is common

LAGE = LBOE = 90°

AO = DB (given)

AD = DB (Siven)

ABED = ABED (SAS)

But LAED = LECB (wresponding L = as DEllac)
and LBED = LEBC (att. L=a, DEllac)
... LECB = LEBC

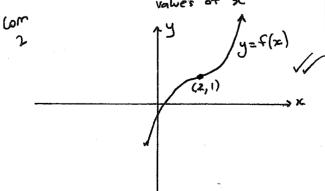
peas

. . BE = EC (=ides opp = Lin on isosceles Acre =)

(c)
$$f(2)=1$$
 \Rightarrow passes through $(2,1)$
 $f'(2)=0$ \Rightarrow stat. point at $(2,1)$
 $f''(2)=0$ \Rightarrow neither concare up or dern ad $(2,1)$

i.e. point of inflexion

f (x) >> 0 => curve increasing or stationary for all values of x



QUESTION 9: (12 marks)

(a) (i)
$$P = \frac{-x^2}{1280} (4x - 240)$$

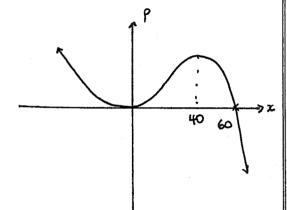
when x = 40,

$$\beta = \frac{-40^2}{1280} (160 - 240) \checkmark$$

Calc

... A shotest completing 40 assignments understands 100% of the course.

(ii)



$$P = -\frac{4x^3}{1260} + \frac{240x^2}{1260}$$
$$= -\frac{x^3}{320} + \frac{3x^2}{11}$$

$$\frac{df}{dx} = \frac{-3x^2}{320} + \frac{6x}{16}$$

Stationary pt $\Rightarrow \frac{dP}{dx} = 0$ $\frac{-3x^2}{320} + \frac{bx}{16} = 0$ $\frac{x}{320} \left(-3x + 120 \right) = 0$ 4x = 0 or 40

of the course undestrood / decreases.

$$\frac{d^2 l}{dx^2} = \frac{-6x}{320} + \frac{3}{8}$$
$$= \frac{-3x}{160} + \frac{3}{8}$$

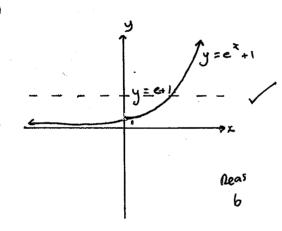
$$\frac{-3x}{160} + \frac{3}{8} = 0$$

$$\frac{3x}{160} = \frac{3}{8}$$

$$x = 20$$

. Increasing at the greatest rate when x = 20.

(b) (i)



(ii)
$$e+1 = e^{x} + 1$$

 $e = e^{x}$
 $x = 1$
 $y = e+1$

(iii)
$$V = \pi \int_{0}^{1} (e+i)^{2} dx - \pi \int_{0}^{1} (e^{x}+i)^{2} dx$$

$$= \pi \left[(e+i)^{2} \times \right]_{0}^{1} - \pi \int_{0}^{1} e^{2x} + 2e^{x} + 1 dx$$

$$= \pi \left((e+i)^{2} - \pi \right) \left[\frac{1}{2} e^{2x} + 2e^{x} + x \right]_{0}^{1}$$

$$= \pi \left((e+i)^{2} - \pi \right) \left[\frac{1}{2} e^{2x} + 2e^{x} + 1 \right]_{0}^{1}$$

$$= \pi \left((e^{2} + 2e + 1) - \frac{1}{2} e^{2x} - 2e^{x} + \frac{3}{2} \right)$$

$$= \pi \left((e^{2} + 2e + 1) - \frac{1}{2} e^{2x} - 2e^{x} + \frac{3}{2} \right)$$

$$= \pi \left((e^{2} + 2e + 1) - \frac{1}{2} e^{2x} - 2e^{x} + \frac{3}{2} \right)$$

$$= \pi \left((e^{2} + 2e + 1) - \frac{1}{2} e^{2x} - 2e^{x} + \frac{3}{2} \right)$$

$$= \pi \left((e^{2} + 2e + 1) - \frac{1}{2} e^{2x} - 2e^{x} + \frac{3}{2} \right)$$

QUESTION 10: (12 marks)

(a) (i)
$$x = 2 + 2\sin \pi_{x0}$$

 $\therefore x = 2$.

when
$$t = 1$$
, $V = \frac{11}{2} \cos \frac{\pi}{4}$

(a^{\c}

$$= \frac{\pi}{2} \times \frac{1}{\sqrt{2}}$$

$$= \frac{\pi}{2\sqrt{2}} \left(\text{or } \frac{\sqrt{2}\pi}{4} \right) \sqrt{\frac{2}{\sqrt{2}}}$$

· . t = 2 , 6, 10, ...

. . Stationary for the second time at t=6.

(b)

(i)

+	1	2	3	4	5	6
1	2	3	4	5	٩	7
2	3	4	5	6	7	8
3	4	5	6	7	8	q
l _k	5	6	7	8	9	10
5	6	7	8	9	lo	11
6	7	8	9	10	11	12

(ii)
$$P(\text{even}, \text{closer to 5}) = \frac{1}{2}$$
 $P(\text{even}, \text{closer to 10}) = \frac{1}{2}$
 $P(\text{odd}, \text{closer to 5}) = \frac{2}{3}$
 $P(\text{odd}, \text{closer to 10}) = \frac{1}{3}$

Player A has a much higher 2 chance of removing 18 blue counters (ty each time) than Player B (to each time)

PROFILES

Reasoning

Calc

$$\begin{array}{c}
(am) \\
Q | (f) - 2 \\
Q + (a) - 3 \\
Q$$